Kuwait University | Math 102 | September 21, 1997
Dept. of Math. and Comp. Sc. | Final (Incomplete) | Duration: 120 minutes

Calculators and mobile phones are not allowed.
Answer all of the following questions.

1. Let $f(x) = \dfrac{3e^x - 2}{e^x + 4}$. Show that f^{-1} exits and find $f^{-1}(x)$. State the domain and range of f^{-1}. (5points)

2. Find $\dfrac{dy}{dx}$, if $y = \dfrac{(1+x^3)^{\sin^{-1}\sqrt{x}}}{e^{\tan^{-1}x}\, e^{\sinh x}}$ (4 points)

3. Find the limit, if it exits: $\lim\limits_{x\to 0+} \left(\dfrac{2^x + 3^x}{2}\right)^{\frac{1}{x}}$. (4 points)

4. Evaluate the following integrals (4 points each)

(a) $\displaystyle\int \frac{\tan^{-1}(\ln x)}{x}\, dx$ (b) $\displaystyle\int (1+\cos x)^{\frac{3}{2}}\, dx$

(c) $\displaystyle\int \sqrt{(x^2+1)\tanh(\ln x)}\, dx$ (d) $\displaystyle\int \frac{3x^3 + 5x^2 + 4x + 2}{x^2(x^2+2x+2)}\, dx.$

5. Determine whether the following integral is convergent or divergent. Find its value, if convergent

$$\int_0^{\frac{\pi}{2}} \frac{dx}{1-\cos x}.$$

(4 points)

6. Find the center, vertices, foci and sketch the graph of the following equation: $16x^2 + 9y^2 + 64x - 18y - 71 = 0$. (5 points)

7. Find the area of the region outside of $r = 3 + 3\cos\theta$ and inside of $r = 9\cos\theta$. (4 points)

8. Find the equation of the tangent line to the curve given parametically by $x(t) = t^2 + t + 1$, $y(t) = \frac{t^3}{3} - \frac{t^2}{2} + 2$, at the point $t = 1$. Also find the points where the tangent line is parallel to the x-axis. (4 points)

9. Find the point of intersection of the lines:

(L_1): $x = 1 + t$, $y = -1 - t$, $z = -4 + 2t$,

(L_2): $x = 1 - u$, $y = 1 + 3u$, $z = 2u$,

if they intersect. (4 points)

Total 50 points